1)

for with

1. Yes, the result is consistent with a second order approximation; decreasing by decreases the error by a factor of , which is appropriate for the error, order .

2)

For with

1. Once again, the result is consistent with a second order approximation; implies the error will shrink with the square of .

3)

2nd order stencil for the 1st derivative:

Thus:

And our stencil is:

4)

b) Depending on what you call large n, yes, the error shrinks appreciably, until

6)

The lower order approximation does better, most likely because we’re dealing with a square wave, so joining line segments approaches the solution faster than curves. Both, predictably, have difficulty with the discontinuous jump at x=0.5, though at 10000 points they start getting very close to vertical,

Optional:

In an image the image intensity drops off rapidly near boundaries, thus one could find the change in intensity and thus detect the areas of most rapid change in intensity and highlight it as the boundary.